Special Relativity

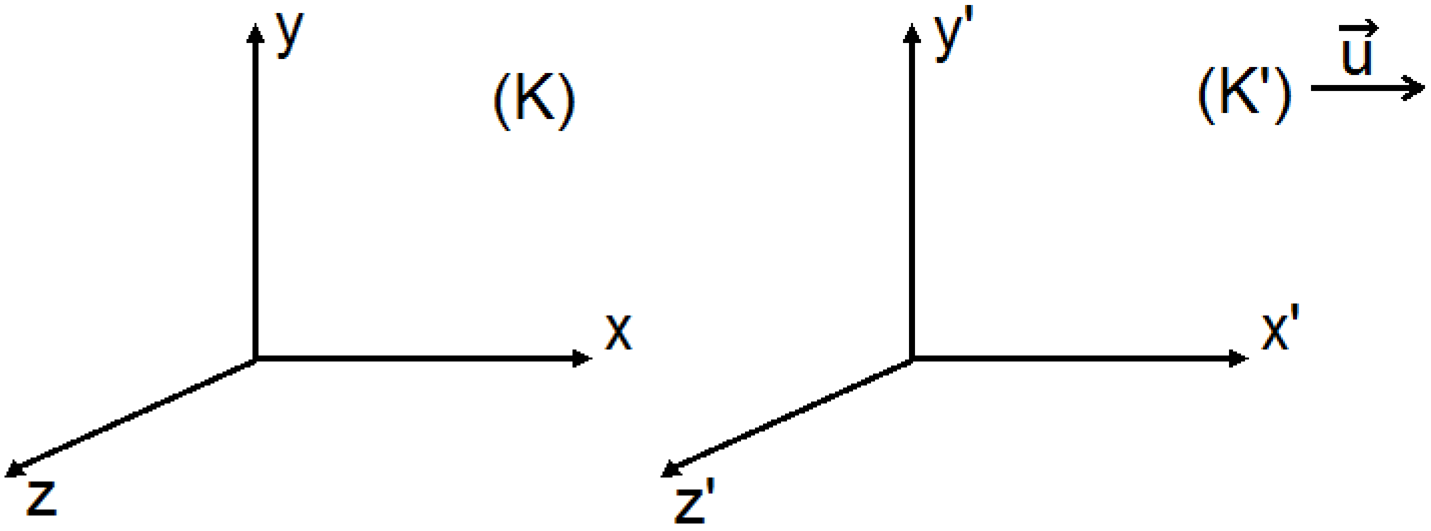
Sébastien Bastide

The Zeitglockenturm

- Connected to the other clocks in Bern
- Attempt to unify time in Switzerland

Scientific context

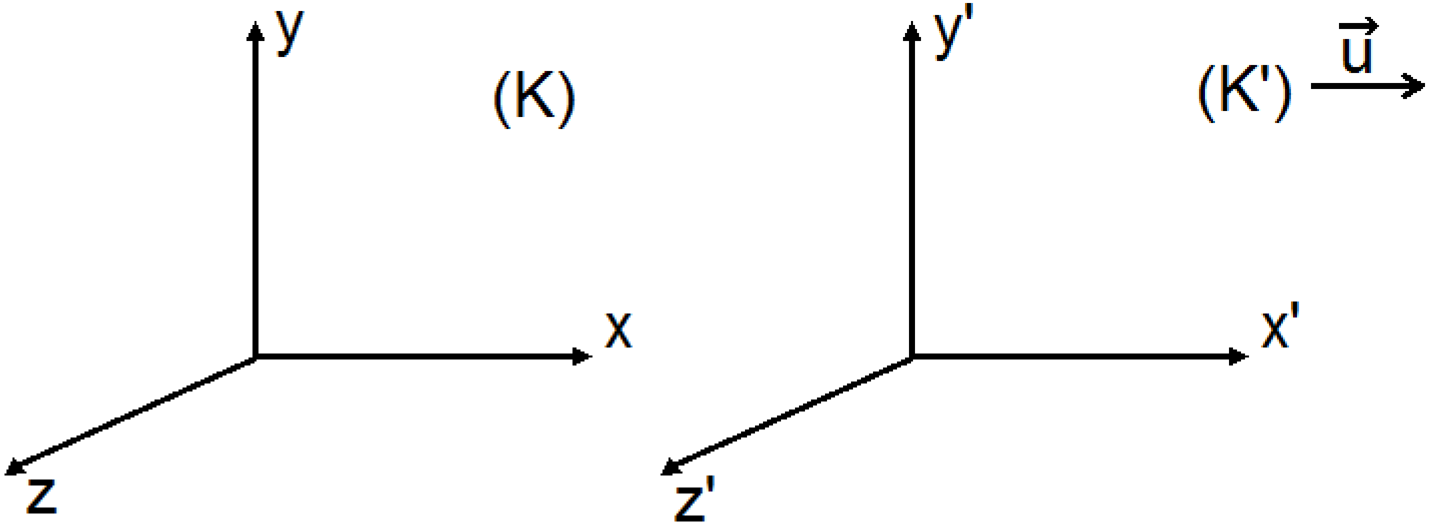
Principle of relativity



The laws of physics have the same form in all inertial frames.

Scientific context

Principle of relativity



$$\begin{cases} t' = t \\ x' = x - ut \\ y' = y \\ z' = z \end{cases} \Rightarrow w = v - u$$

Galilean transformation

Scientific context

Maxwell's Equations

$$\nabla \cdot \mathrm{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \times \mathrm{E} = -\frac{\partial \mathrm{B}}{\partial \mathrm{t}}$$

$$\nabla \cdot \mathrm{B} = 0$$

$$\nabla \times \mathrm{B} = \mu_{0\mathrm{J}} + \mu_0 \epsilon_0 \frac{\partial \mathrm{E}}{\partial \mathrm{t}}$$

Electromagnetic waves travel through the Luminiferous Ether

Problem ?

- Maxwell's equations are not invariant by the Galilean transformation
- Incompatibility of the composition law for velocities and the constancy of the speed of light

Lorentz transformation

Named after Hendrik Lorentz

$$\begin{cases} t' = \gamma\left(t - \frac{xu}{c^2}\right) \\ x' = \gamma(x - ut) \\ y' = y \\ z' = z \end{cases} \quad \text{with} \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Composition law for velocities

$$w = \frac{v - u}{1 - \frac{vu}{c^2}}$$

Consequences of the Lorentz transformation

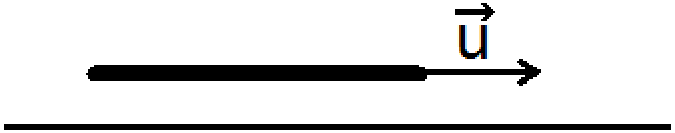
Length contraction

$$\Delta x' = \gamma(\Delta x - u\Delta t)$$

$L_0 = \gamma L$ for $\Delta t = 0$

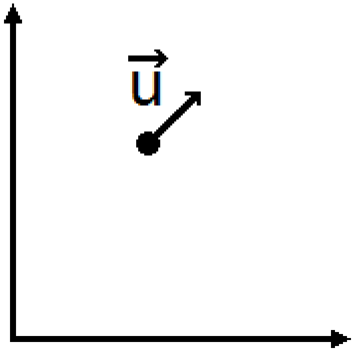
L_0 is called the proper length

$$L = \frac{L_0}{\gamma}$$



Consequences of the Lorentz transformation

Time dilatation



$$\Delta t' = \gamma\left(\Delta t - \frac{u \Delta x}{c^2}\right)$$

$\Delta t = \gamma \Delta \tau$ for $\Delta x = 0$

τ is called proper time

We can derive $\frac{dt}{d\tau} = \gamma$

Consequences of the Lorentz transformation

Simultaneity

$$\Delta t' = \gamma\left(\Delta t - \frac{u\Delta x}{c^2}\right)$$

$$\Delta t' = -\gamma \frac{u\Delta x}{c^2} \neq 0$$ for $\Delta t = 0$

unless $\Delta x = 0$ (Concomitant events)

Consequences of the Lorentz transformation

Introduction to four-vectors

Four-position: $$X = \left(\underset{ct}{x_0}, \underset{x}{x_1}, \underset{y}{x_2}, \underset{z}{x_3}\right)$$

Four-velocity: $$V = \frac{dX}{d\tau} = \frac{dX}{dt}\frac{dt}{d\tau} = \gamma\left(\frac{dx_0}{dt}, \frac{dx_1}{dt}, \frac{dx_2}{dt}, \frac{dx_3}{dt}\right)$$

Four-momentum: $$P = m_0 V = \left(\frac{m_0 c}{\sqrt{1-\frac{v^2}{c^2}}}, \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

Consequences of the Lorentz transformation

Mass-energy equivalence

Taylor series of $\left(1+x\right)^n$ when $x \rightarrow 0$

$$\left(1+x\right)^n = 1+nx+...$$

$$cP_0 = m_0 c^2 \left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} = m_0 c^2 + \frac{1}{2} m_0 v^2 + \frac{3}{8} m_0 \frac{v^4}{c^2} + ...$$

Conclusion

- No need to abandon the galilean principle of relativity or the constancy of the speed of light
- Time and space are combined in a single interwoven continuum
- Length, time and simultaneity are not absolute